ARJUNA

JEE

LECTURE NUMBER - 9

Basic Math

MSM SIR

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TABLE OF CONTENT

01 TWO IMPORTANT MODULUS RELATIONS

02 LOGARITHM STARTS

Maths mai bahut Kaam aate hain

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RECAP

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① $|x| \leq 2$

$\Rightarrow \quad x \in [-2, 2]$

OR

$-2 \leq x \leq 2$

② $|E| \geq 2$

$\Rightarrow \quad E \geq 2$ OR $E \leq -2$

(3) Solve for $x \rightarrow$

$|x| \geq -2$

P, Z

always TRUE; $x \in R$

Ans.

(4) Solve for $x -$

$|x| \leq -2$

P, Z

False (Invalid)

$x \in \phi$ Ans

⑤ Solve for $x \rightarrow$

$$|x| > -7$$

$|x|$

P, Z

Valid

$x \in (-\infty, \infty)$

OR

$x \in R$

Ans

⑥ Solve for $x \rightarrow$

$$|x| < -10$$

$|x|$

P, Z

Invalid

$x \in \phi$ Ans.

TWO IMPORTANT MODULUS RELATIONS

Relation 1 → If $|E_1| + |E_2| = |E_1 + E_2|$

Then $E_1 \cdot E_2 \geq 0$

Relation 2 → If $|E_1| + |E_2| = |E_1 - E_2|$

Then $E_1 \cdot E_2 \leq 0$.

Q. Solve for x :

$|x+1| + |x-1| = |2x|$

$E_1 = x+1$

$E_2 = x-1$

$E_1 + E_2 = (x+1) + (x-1)$

$= 2x$

$$|E_1| + |E_2| = |E_1 + E_2|$$

$$\Rightarrow E_1 \cdot E_2 \geq 0$$

$$(x+1)\cdot(x-1) \geq 0$$

$x \in (-\infty, -1] \cup [1, \infty)$ Ans.

Q. Solve for x :

$|x+1| + |x-1| = 2$

✓ $E_1 = x+1$

✓ $E_2 = x-1$

$E_1 - E_2 = (x+1) - (x-1) = 2$

$|E_1| + |E_2| = |E_1 - E_2|$?

$\Rightarrow E_1 \cdot E_2 \leq 0$

$(x+1)(x-1) \leq 0$

$x \in [-1, 1]$ Ans

Gyaan →

$$|E| = |-E|$$

- $|x-2| = |-(x-2)| = |2-x|$

  ($x-2 = E$, $-(x-2) = -E$)

- $|x^2-5x+6| = |-(x^2-5x+6)|$

  ($x^2-5x+6 = E$)

  $= |5x-x^2-6|$.

SOLVE FOR 'x' :

(1) $|x-3|+|x-5|=|8-2x|$

(2) $|x^2-2x|+|3x-6|=|5x-6-x^2|$

(3) $|x^2-4|+|x^2-9|=|2x^2-13|$

(4) $|1-\frac{|x|}{1+|x|}| \geq \frac{1}{2}$

① $E_1 = x-3$

$E_2 = x-5$

$E_1+E_2 = 2x-8$

$|E_1+E_2| = |2x-8|$

$= |-(2x-8)|$

$= |8-2x|$

$|E_1|+|E_2| = |E_1+E_2|$

$E_1 \cdot E_2 \geq 0$

$(x-3)(x-5) \geq 0$

$x \in (-\infty, 3] \cup [5, \infty)$ Ans.

SOLVE FOR 'x' :

(1) $|x-3|+|x-5|=|8-2x|$

(2) $|x^2-2x|+|3x-6|=|5x-6-x^2|$

(3) $|x^2-4|+|x^2-9|=|2x^2-13|$

(4) $|1-\frac{|x|}{1+|x|}| \geq \frac{1}{2}$

$|E_1-E_2|$

(2) $E_1 = x^2-2x$

$E_2 = 3x-6$

$E_1+E_2 = x^2+x-6$

$|E_1+E_2| = |x^2+x-6| = |E|$

$= |-(x^2+x-6)| = |-E|$

$= |-x^2-x+6|$

Not $|E_1+E_2|$

$|E_1-E_2|$

$= |x^2-2x-(3x-6)|$

$= |x^2-5x+6|$

$= |-(x^2-5x+6)| = |5x-x^2-6|$

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(2) $|E_1| + |E_2| = |E_1 - E_2|$

$$E_1 \cdot E_2 \leq 0$$

$$(x^2 - 2x)(3x - 6) \leq 0$$

$$x(x-2)\,3(x-2) \leq 0$$

$$3x(x-2)^2 \leq 0$$

No-Ne,

CB, AP ($x = 2$)

$x \leq 0$

$x \in (-\infty, 0] \cup \{2\}$ Ans

SOLVE FOR 'x' :

(1) $|x-3|+|x-5|=|8-2x|$

(2) $|x^2-2x|+|3x-6|=|5x-6-x^2|$

(3) $|x^2-4|+|x^2-9|=|2x^2-13|$

(4) $\left|1-\frac{|x|}{1+|x|}\right| \geq \frac{1}{2}$

PW

(3) $E_1 = x^2-4$

$E_2 = x^2-9$

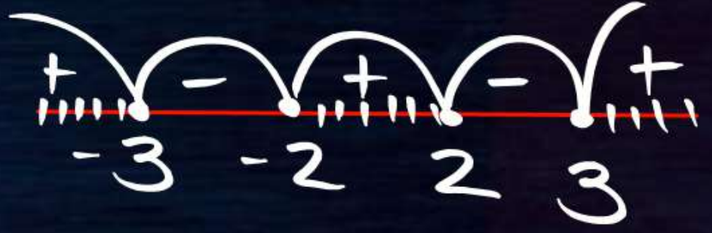
$E_1+E_2 = 2x^2-13$

$|E_1|+|E_2| = |E_1+E_2|$

$E_1 \cdot E_2 \geq 0$

$(x^2-4)(x^2-9) \geq 0$

$(x+2)(x+3)(x-2)(x-3) \geq 0$



Ans →

$x \in (-\infty, -3]$

$\cup [-2, 2]$

$\cup [3, \infty)$

PROOF →

- ✓ $E_1 = 2$, $E_2 = 3$

$|E_1| + |E_2| = |E_1 + E_2|$

$2 + 3 = 5$

$E_1 \cdot E_2 > 0$ ✓

- ✓ $E_1 = 7$, $E_2 = 0$

$|E_1| + |E_2| = |E_1 + E_2|$ ✓

$|7| + |0| = |7 + 0|$

$E_1 \cdot E_2 = 0$

- $E_1 = 2$, $E_2 = -3$

$|E_1| = 2$

$|E_2| = |-3| = 3$

$|E_1 + E_2| = |2 - 3| = 1$

$|E_1| + |E_2| \neq |E_1 + E_2|$ ✓

$E_1 \cdot E_2 < 0$

- $E_1 = -2$, $E_2 = -3$

$|E_1| + |E_2| = |-2| + |-3| = 5$

$|E_1 + E_2| = |-2 - 3| = 5$

$|E_1| + |E_2| = |E_1 + E_2|$

$E_1 \cdot E_2 > 0$

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Conclusion → $|\epsilon_1| + |\epsilon_2| = |\epsilon_1 + \epsilon_2|$

Then

$$\epsilon_1 \cdot \epsilon_2 \geq 0$$

PROOF →

- $\varepsilon_1 = 2;\ \varepsilon_2 = 3$

$|\varepsilon_1| + |\varepsilon_2| = 5$

$|\varepsilon_1 - \varepsilon_2| = |2-3| = 1$

$|\varepsilon_1| + |\varepsilon_2| \neq |\varepsilon_1 - \varepsilon_2|$

$\varepsilon_1 \cdot \varepsilon_2 > 0$ ✗

- $\varepsilon_1 = 2,\ \varepsilon_2 = -7$

$|\varepsilon_1| = 2;\ |\varepsilon_2| = 7$

$|\varepsilon_1| + |\varepsilon_2| = 9$

$|\varepsilon_1 - \varepsilon_2| = |2-(-7)| = 9$

$|\varepsilon_1| + |\varepsilon_2| = |\varepsilon_1 - \varepsilon_2|$

$\varepsilon_1 \cdot \varepsilon_2 < 0$ ✓

- $\varepsilon_1 = -2,\ \varepsilon_2 = -3$

$|\varepsilon_1| + |\varepsilon_2| = 5$

$|\varepsilon_1 - \varepsilon_2| = |-2+3| = 1$

$|\varepsilon_1| + |\varepsilon_2| \neq |\varepsilon_1 - \varepsilon_2|$

$\varepsilon_1 \cdot \varepsilon_2 > 0$ ✗

- $\varepsilon_1 = -7,\ \varepsilon_2 = 0$

$|\varepsilon_1| + |\varepsilon_2| = 7 = |\varepsilon_1 - \varepsilon_2|$

$\varepsilon_1 \cdot \varepsilon_2 = 0$ ✓

Conclusion →

$$|E_1| + |E_2| = |E_1 - E_2|$$

Then

$$E_1 \cdot E_2 \leq 0$$

common language ≡ "Log"

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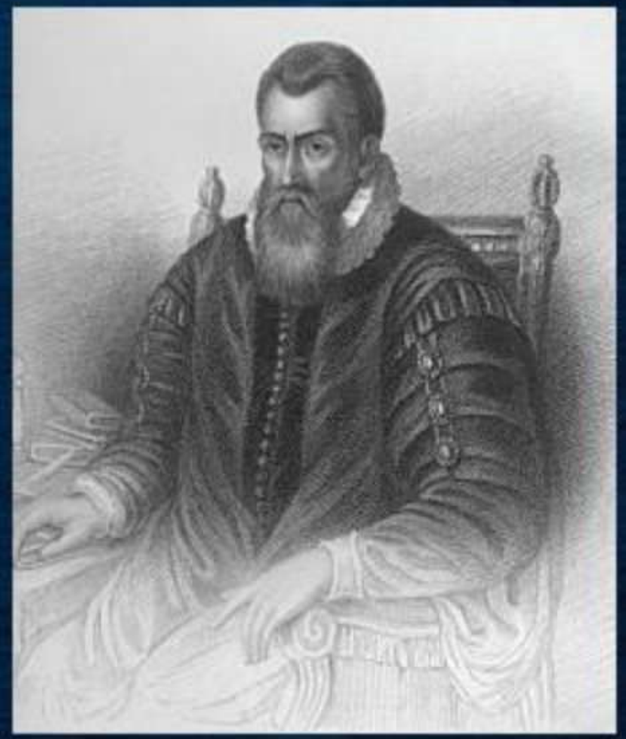
"Calculus"

80% Questions will involve theory of log.



"JOHN NAPIER"

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EVERY POSITIVE REAL NUMBER 'N' CAN BE EXPRESSED IN EXPONENTIAL FORM.
i.e. $N=a^L$. (where $a > 0$, $a \neq 1$)

$N = (-2)^L$ ← $a = -2$

$L = 1$

$N = -2$

$L = 2$

$N = (-2)^2 = 4$

$L = 3$

$N = -8$

$a = 0$

$N = (0)^L$ → $L = 1$ → $N = 0$ (But $N > 0$)

$a = 1$

$N = (1)^L = 1$

$$N = (a)^{L}$$

$(N > 0)$

$a > 0, a \neq 1$

"LEARN"

DEFINITION:-

$N = (a)^{L}$ → log

N → Number

a → Base

L → Kuch bhi

$(N>0;\ a>0;\ a\neq 1)$

$\Downarrow$

$$L = \log_a N$$

$$L = \log_a N \iff N = a^L$$

$a > 0 ; a \neq 1$

$N > 0$

This is logarithm of 'N' whose Base is 'a'

Q. Find 'L' if $L = \log_2 16$.

$L = \log_a N$

$N = 16, a = 2$

$N = a^L$

$16 = (2)^L \Rightarrow 2^4 = 2^L$

$\Rightarrow \boxed{L = 4}$ Ans.

JAROORI BAATEIN

① $(\log_a N) = L \implies \left.\begin{matrix} a>0 \\ a\neq 1 \\ N>0 \end{matrix}\right\}$ ∩ ★

→ "Domain of Log" ★

"

{log Kab Defined ∀ |

"

{log Kab meaningful ∀ |

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Q. If $\log_x(2x-1)$ is meaningful, find x.

$N = 2x-1$, $a = x$

$N > 0$, $a > 0$, $a \neq 1$ $\cap$

$2x-1 > 0$, $x > 0$, $x \neq 1$ $\cap$

$\cap \begin{cases} x \in \left(\frac{1}{2}, \infty\right) \\ x \in (0, \infty) \end{cases}$

finally exclude "1"

$x \in \left(\frac{1}{2}, \infty\right) - \{1\}$

Ans

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Q→ Find the Domain of −

$$\log_{(x-1)}(2x-1) = L.$$

Soln →

$N = 2x-1$

$a = x-1$

$2x-1>0 \rightarrow x \in \left(\frac{1}{2}, \infty\right)$

$x-1>0 \rightarrow x \in (1, \infty)$

$\cap$

$a \neq 1 \Rightarrow x-1 \neq 1$

$x \neq 2$

$$x \in (1,\infty) - \{2\}$$

Ans

COMMON AND NATURAL LOG

Common log means Base is 10. $a = 10$

$$\log N = \log_{10} N$$

Used in Physics and Chemistry

Natural log means Base is 'e'. $a = e$

$$\ln N = \log_e N$$

Used in maths

$e \simeq 2.71$ ★

↳ Napierian Base

ex:-

$\log 2 = \log_{10} 2$

$\ln 2 = \log_e 2$.

PROPERTIES OF LOGARITHM

Property – 1 $\Rightarrow$

Extra class
Saturday
4:15 pm

Finish previous two HW
DPP1 to DPP5

Revise aaj K notes
Pen and Paper

JBK... JBK...
JBK

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JBK...JBK...JBK

- TO *FUTURE IIT-IANS*